Motion in Two Dimensions

Projectiles

What is a projectile?

- A **projectile** is any object that has been put in motion by some means and continues to move while gravity changes its motion.
  - We don't care why or how it initially moved; the only thing that matters is **only gravity** is affecting the motion at this time.

Examples:

- Examples of projectiles include:
  - A football after it has been thrown or kicked.
  - A skydiver in free fall.
  - A cheerleader or gymnast who is in the air.
  - An arrow shot from a bow.
  - Cannonball shot from a cannon.

Two-Dimensional Motion

- Projectiles move in two dimensions
  - There is a horizontal component (piece) to its motion.
  - There is also a vertical component (piece) to its motion.

  - You must treat the horizontal and vertical motions independent from one another. THEY HAVE NO EFFECT ON EACH OTHER.

Two-Dimensional Motion

- The horizontal component of a projectile's motion experiences no change since there are no horizontal forces present (neglecting air resistance).

- This means there is no horizontal acceleration. $a_x = 0\ m/s^2$
- This also means the horizontal velocity is constant. v_x **is constant**

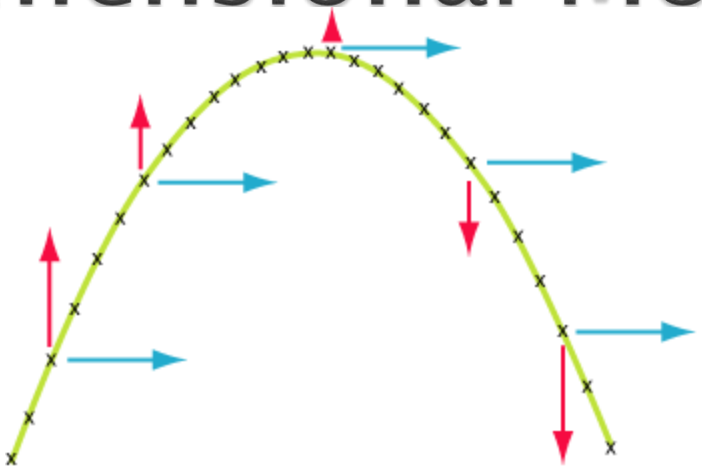
Two-Dimensional Motion

- The vertical component of the projectile's motion is changing because the force of gravity acts straight down ALWAYS.

- There is a vertical acceleration (free-fall).
  - $a_y = -9.8\ m/s^2$
- The vertical velocity will increase or decrease depending on the direction it is moving at the time.
  - If the velocity and acceleration are same direction, then the vertical velocity (v_y) will increase.
  - If the velocity and acceleration are in opposite directions, then the vertical velocity (v_y) will decrease.

Two-Dimensional Motion

*The arrow length represents relative velocity.



Notice how the horizontal component of the velocity never changes during the trajectory.

We see that the vertical component of the velocity changes the entire time due to the effect of gravity.

Two-Dimensional Motion

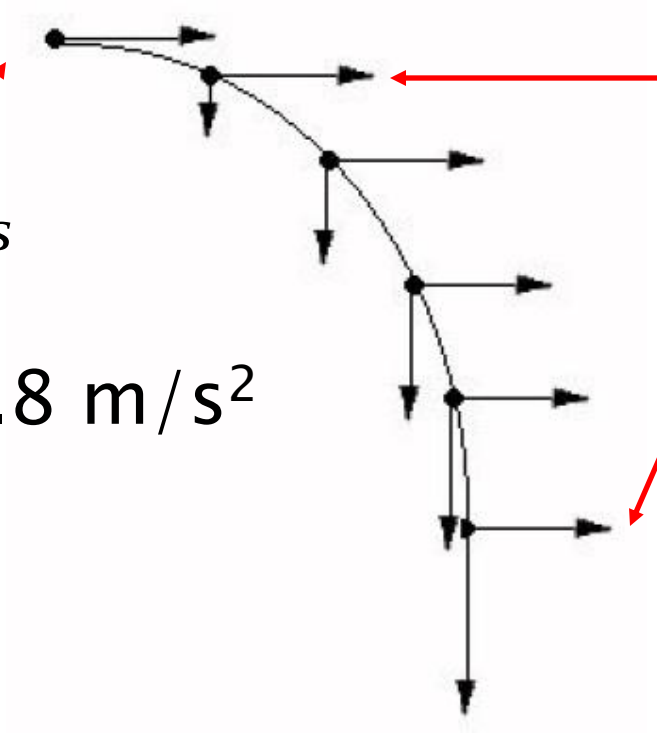
- To summarize the horizontal and vertical components of velocity:

Component	Magnitude	Direction
Horizontal	Constant	Constant
Vertical	Changes	Changes

Remember: Magnitude is another way of saying amount or how much.

Horizontally Launched Projectiles

- These are projectiles that have no initial vertical component of velocity. ($v_{yo} = 0$ m/s)



$v_{oy} = 0 m/s$

$v_{ox} = v_x = \text{constant}$

$a_y = -9.8$ m/s^2

$a_x = 0$ m/s^2

Horizontally Launched Projectiles

- What is happening to the vertical component of the projectile's velocity once it is launched? Explain why this is happening.

Cause and Effect for Horizontally Launched Projectiles

- What factor(s) affect the time that the projectile is in the air?
  - The height, vertical displacement (Δy), determines how long the projectile is in the air.
  - A larger height means it must fall further with the same vertical acceleration so it takes more time.

- What factors determine the horizontal range of the projectile?
  - The time in the air and the horizontal velocity.
  - Therefore, increasing the height will give a larger time and larger horizontal displacement
  - Launching with a greater horizontal velocity will also allow the object to cover more distance in the same time.

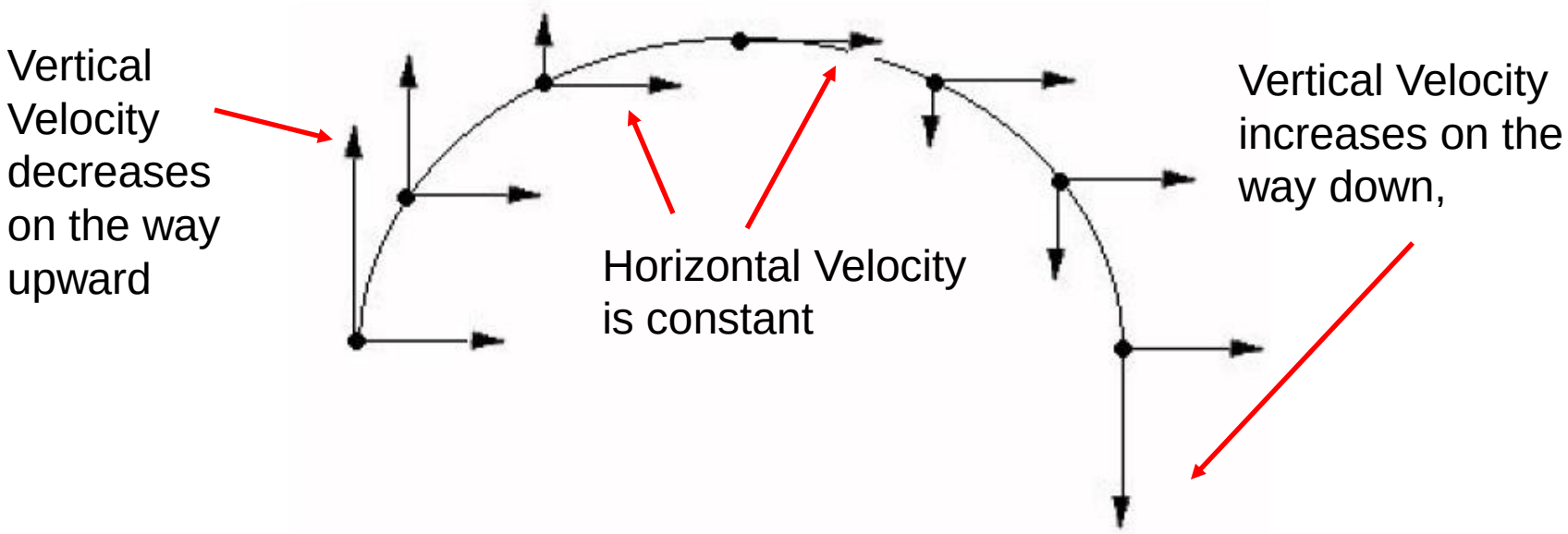
- What determines the vertical velocity when it reaches the ground?
  - The height, vertical displacement, determines how long the object falls.
  - Since it falls longer from greater heights, it also is able to accelerate for a longer time.
  - Thus giving a larger vertical velocity for greater heights.

Projectiles Launched at an Angle

- Projectiles launched at an angle have both an initial horizontal velocity and an initial vertical velocity.
  - The horizontal velocity is still constant.
  - The vertical velocity is changing due to gravity.
    - If the angle is above the horizon, then the initial vertical velocity is positive.

Projectiles Launched at an Angle

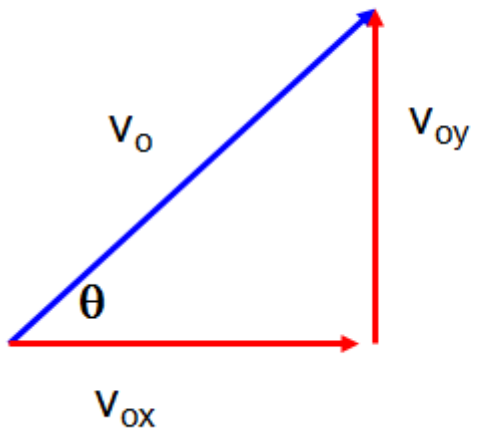
NO Vertical Velocity at the top of the trajectory.



Component	Magnitude	Direction
Horizontal	Constant	Constant
Vertical	Decreases up, 0 @ top, Increases down	Changes

Projectiles Launched at an angle

- The initial velocity has both a horizontal and vertical component that we must resolve before doing the problem.



$$v_{ox} = v_o \cos\theta$$

$$v_{oy} = v_o \sin\theta$$

Projectiles Launched at an angle

- If the projectile launches and lands at the same level, then
  - It has the same velocity at landing as launch.
  - Half of the time is spent rising, half is spent falling
  - The highest point is it the halfway point.