Graphical Analysis of Motion

AP Physics C

Slope – A basic graph model

A basic model for understanding graphs in physics is *SLOPE*.

$$Slope = \frac{Rise}{Run} \; or \; \frac{\Delta y}{\Delta x}$$

$$y = mx + b$$

Using the model - **Look at the formula for velocity.**

$$slope = \frac{Rise}{Run} \Leftrightarrow \bar{v} = \frac{\Delta x}{\Delta t}$$

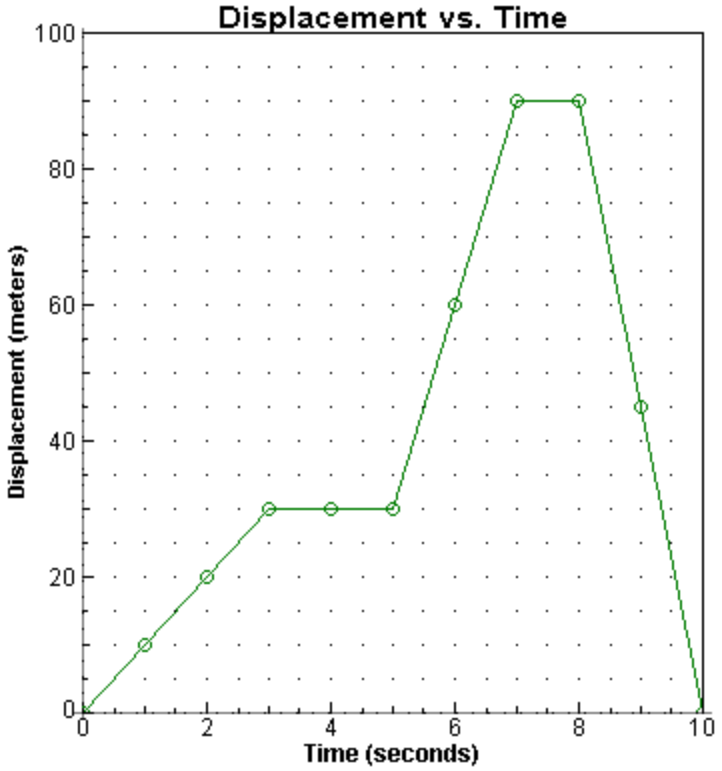
Who gets to play the role of the slope? **Velocity**

Who gets to play the role of the y-axis or the rise? **Displacement**

Who get to play the role of the x-axis or the run? **Time**

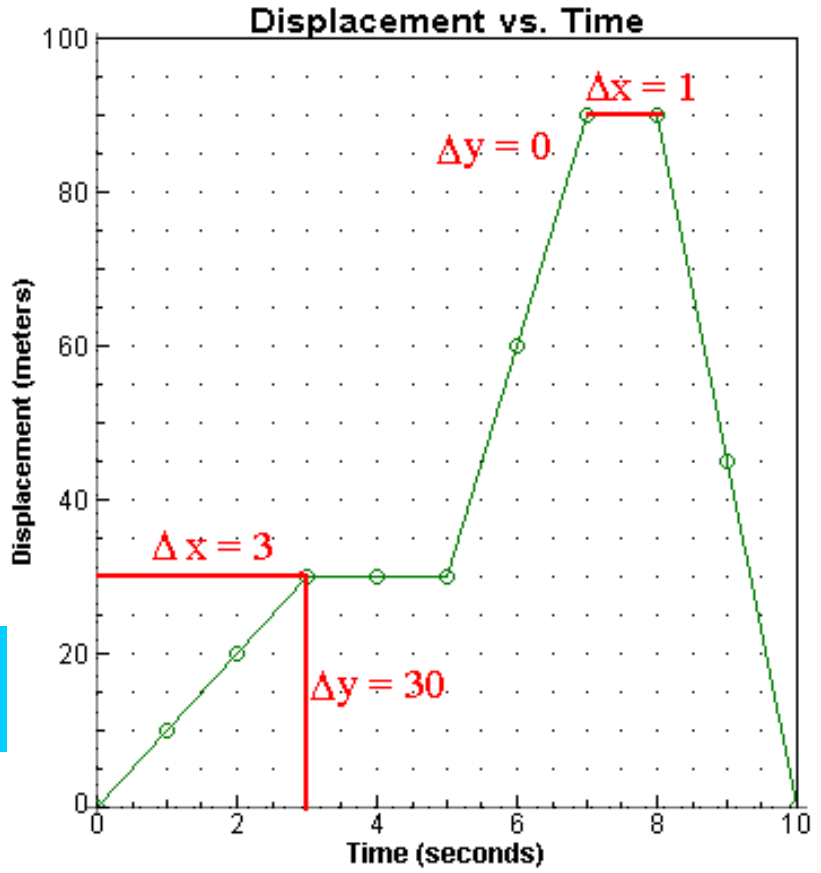
What does all the mean? **It means that if your are given a *Displacement vs. Time* graph, to find the velocity of an object during specific time intervals simply find the slope.**

Displacement vs. Time graph



What is the *velocity* of the object from 0 seconds to 3 seconds?

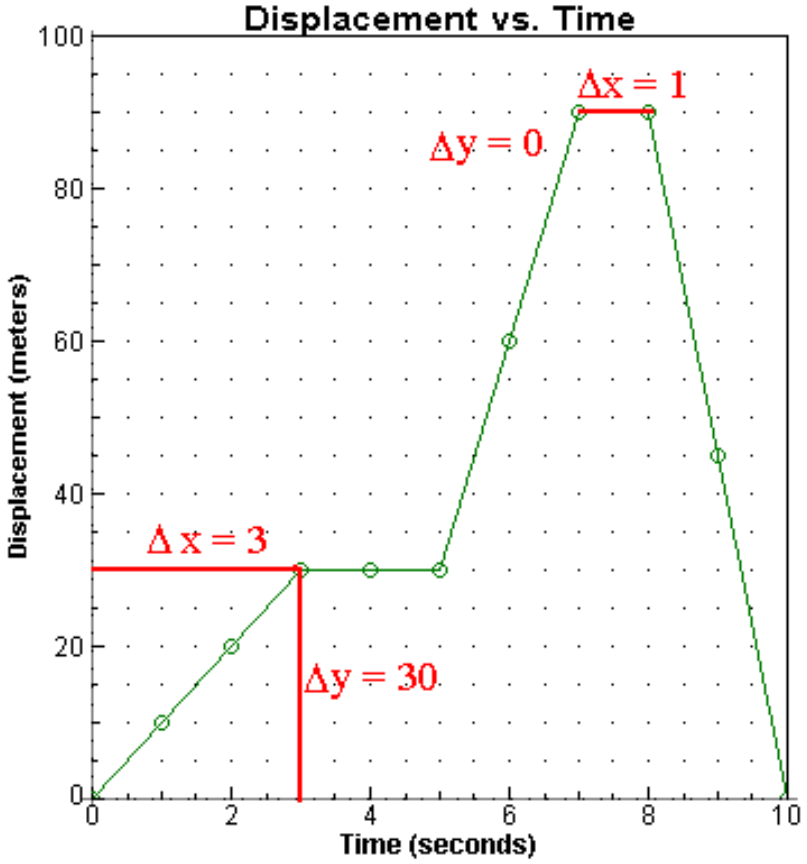
The velocity is the slope!



$$\frac{\Delta y}{\Delta x} = slope = velocity = \frac{30-0}{3-0} = 10 m/s$$

Displacement vs. Time graph

What is the *velocity* of the object from 7 seconds to 8 seconds? Once again...find the slope!

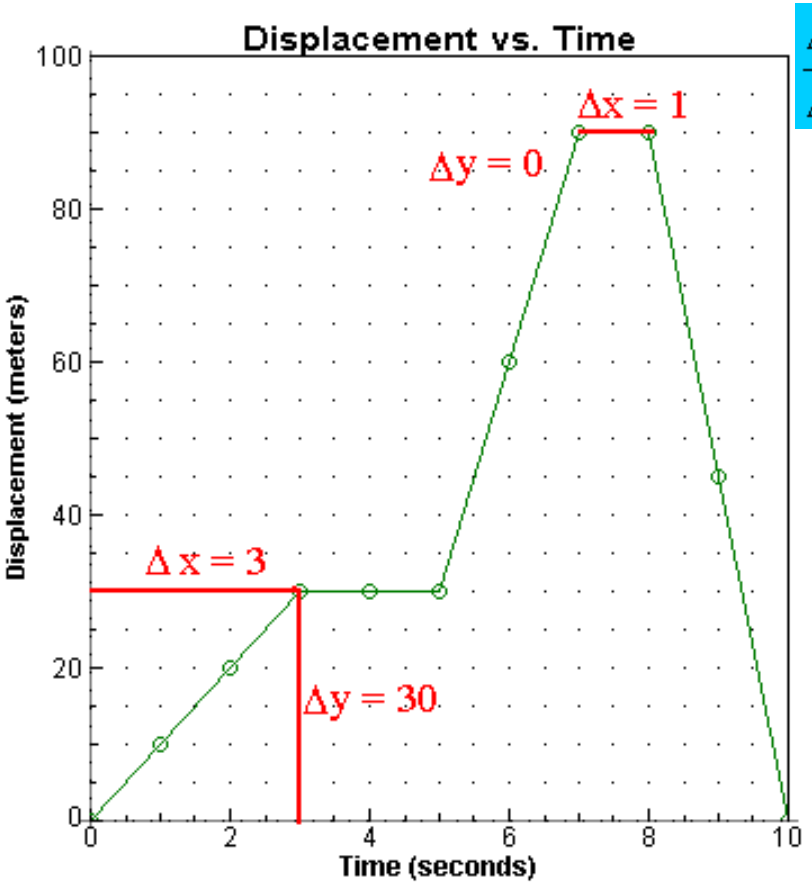


$$\frac{\Delta y}{\Delta x} = slope = velocity = \frac{90-90}{8-7} = 0 m/s$$

A velocity of 0 m/s. What does this mean? It is simple....the object has simply stopped moving for 1 second.

Displacement vs. Time graph

What is the *velocity* from 8-10 seconds? You must remember! To find the *change* it is final - initial.



$$\frac{\Delta y}{\Delta x} = slope = velocity = \frac{0-90}{10-8} = \frac{-90}{2} = -45 m/s$$

The answer is negative! It is no surprise, because the slope is considered to be negative.

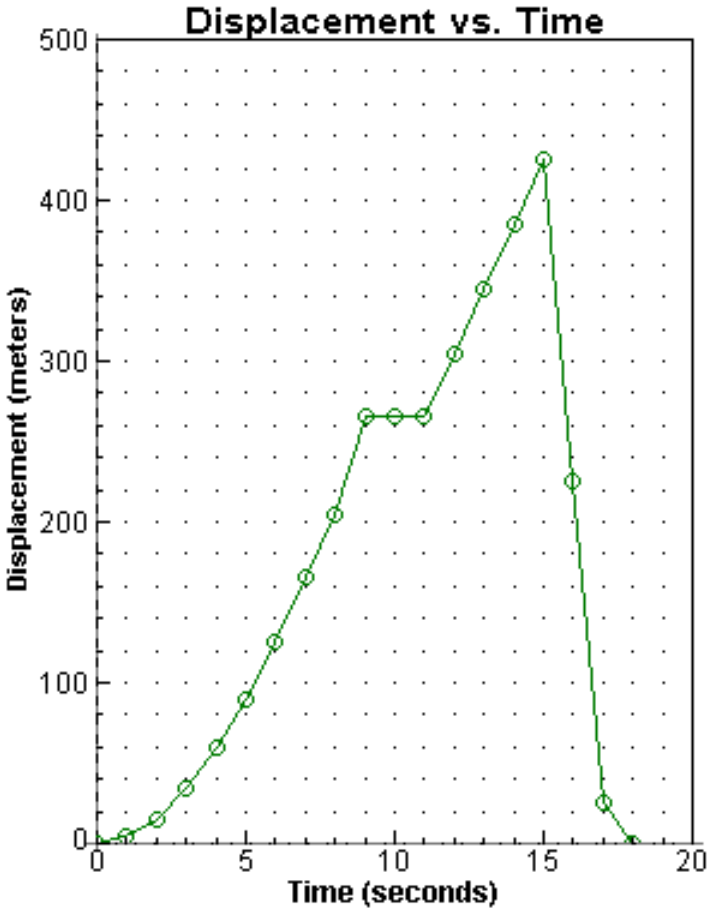
This value could mean several things: The object could be traveling WEST or SOUTH. The object is going backwards - this being the more likely choice!

You should also understand that the slope does NOT change from 0-3s , 5 to 7s and 8-10s.

This means that the object has a CONSTANT VELOCITY or IT IS NOT ACCELERATING.

Example

It is very important that you are able to look at a graph and explain it's motion in great detail. These graphs can be very conceptual.



Look at the time interval t = 0 to t = 9 seconds. What does the slope do?

It increases, the velocity is increasing

Look at the time interval t = 9 to t = 11 seconds. What does the slope do?

No slope. The velocity is ZERO.

Look at the time interval t = 11 to t = 15 seconds. What does the slope do?

The slope is constant and positive. The object is moving forwards at a constant velocity.

Look at the time interval t = 15 to t = 17 seconds. What does the slope do?

The slope is constant and negative. The object is moving backwards at a constant velocity.

Slope – A basic graph model

Let's look at another model

$$Slope = \frac{Rise}{Run}$$

$$Acceleration = \frac{Velocity}{Time} = \frac{\Delta v}{\Delta t}$$

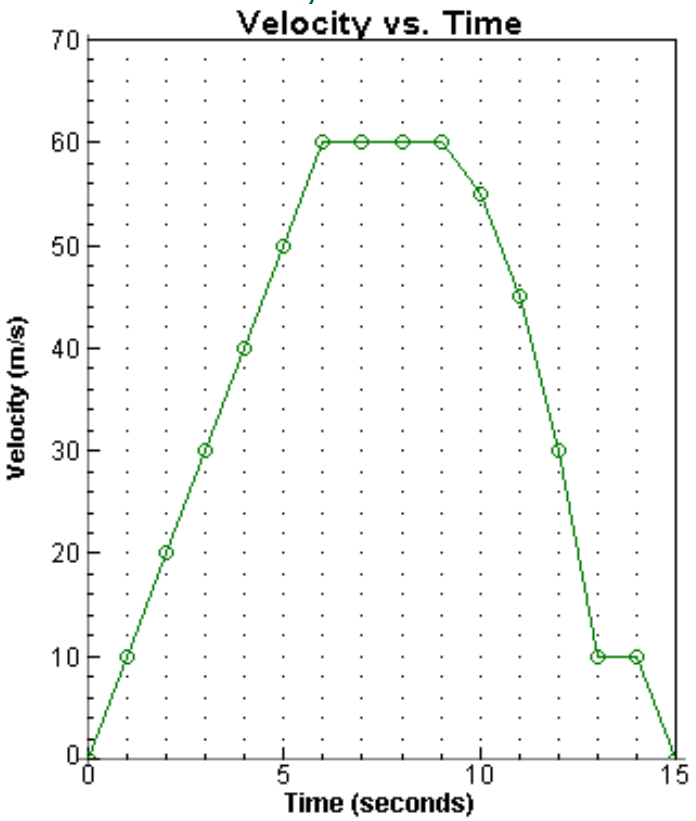
Who gets to play the role of the slope? **Acceleration**

Who gets to play the role of the y-axis or the rise? **Velocity**

Who get to play the role of the x-axis or the run? **Time**

What does all the mean? **It means that if your are given a *Velocity vs. Time* graph. To find the acceleration of an object during specific time intervals simply find the slope.**

Velocity vs. Time Graph



What is the acceleration from 0 to 6s?

$$\frac{\Delta y}{\Delta x} = slope = acceleration = \frac{60-0}{6-0} = 10 m/s/s$$

What is the acceleration from 6 to 9s?

$$\frac{\Delta y}{\Delta x} = slope = acceleration = \frac{60-60}{9-6} = 0 m/s/s$$

You could say one of two things here:

The object has a ZERO acceleration

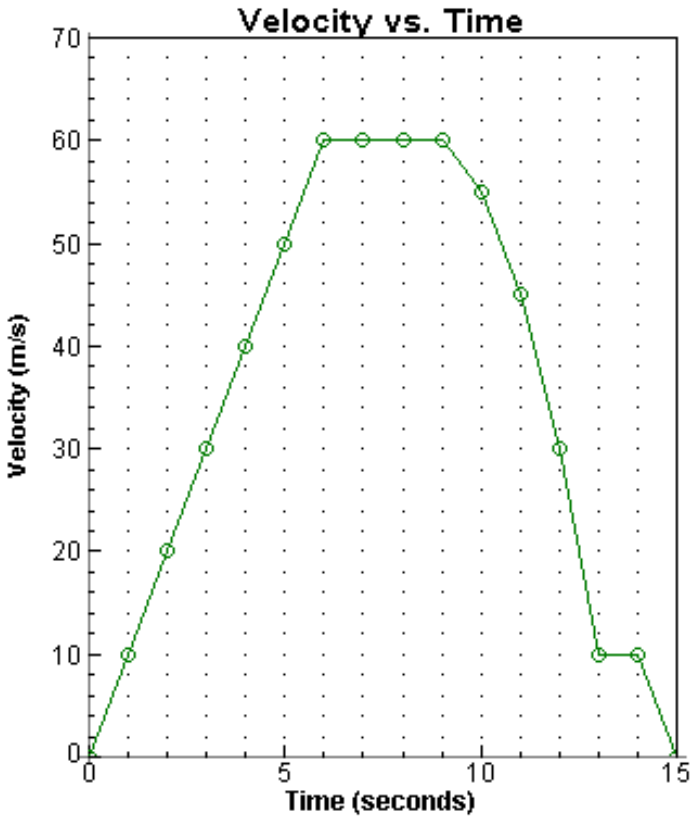
The object has a CONSTANT velocity

What is the acceleration from 14 to 15s?

$$\frac{\Delta y}{\Delta x} = slope = acceleration = \frac{0-10}{15-14} = -10 m/s/s$$

A negative acceleration is sometimes called DECELERATION. In other words, the object is **slowing down**. An object can also have a negative acceleration if it is **falling**. In that case the object is speeding up. CONFUSING? Be careful and make sure you understand **WHY** the negative sign is there.

Velocity vs. Time Graph



Conceptually speaking, what is the object doing during the time interval t = 9 to t = 13 seconds?

Does the steepness or slope increase or decrease?

The slope INCREASES!

According to the graph the slope gets steeper or increases, but in a negative direction.

What this means is that the velocity slows down with a greater *change* each second. The deceleration, in this case, get larger even though the velocity decreases.

The velocity goes from 60 to 55 (a change of 5), then from 55 to 45 (a change of 10), then from 45 to 30 (a change of 15), then from 30 to 10 (a change of 20). Do you see how the change gets LARGER as the velocity gets SMALLER?

Area – the "other" basic graph model

Another basic model for understanding graphs in physics is *AREA*.

$$Area = basexheight$$

$$or$$

$$A = bh$$

Let's try to algebraically make our formulas look like the one above. We'll start with our formula for velocity.

$$A = bh$$

$$Velocity = \frac{Displacement}{Time} \; or$$

$$Displacement = TimexVelocity$$

$$\Delta x = \Delta tv$$

Who gets to play the role of the base?

Time

Who gets to play the role of the height?

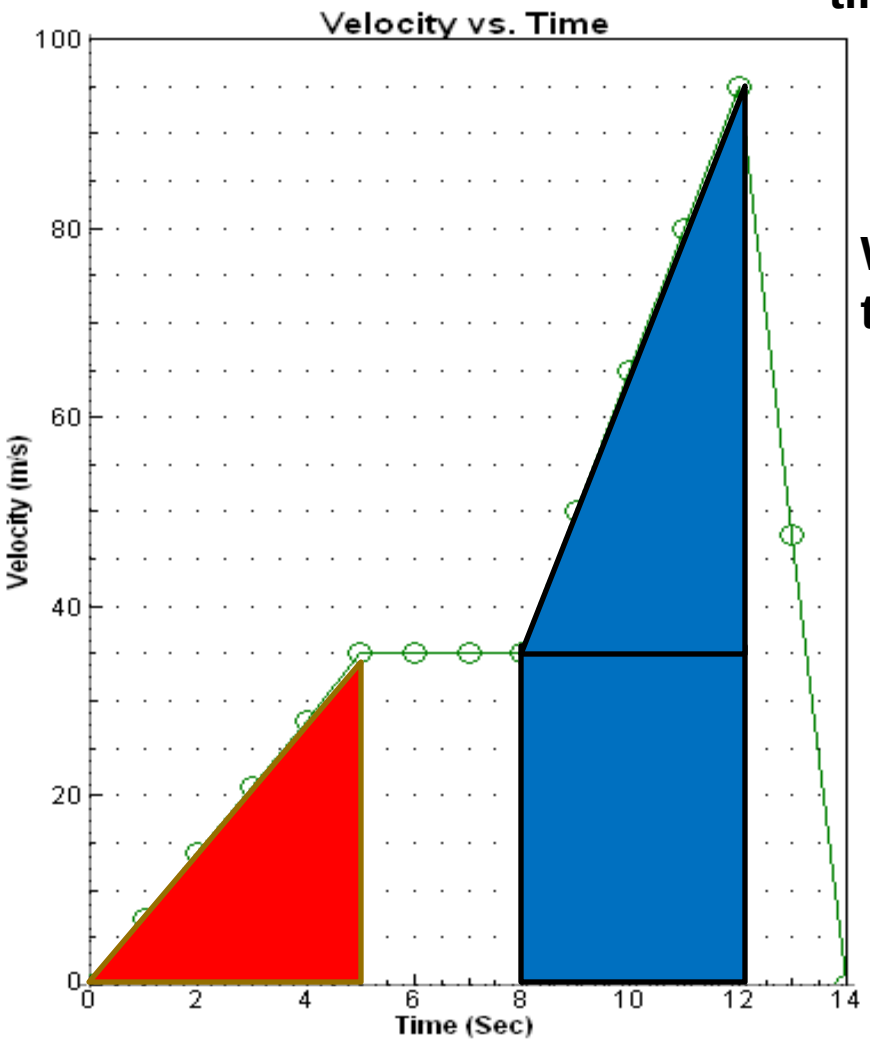
Velocity

What kind of graph is this?

A Velocity vs. Time graph (velocity = y-axis & time = x-axis)

Who gets to play the role of the Area? **Displacement**

Example



What is the displacement during the time interval t = 0 to t = 5 seconds?

That happens to be the area!

$$\tfrac{1}{2}Bh = Area = Displacement = \tfrac{1}{2}(5)(35) = 87.5m$$

What is the displacement during the time interval t = 8 to t = 12 seconds?

Once again...we have to find the area.

During this time period we have a triangle AND a square. We must find the area of each section then ADD them together.

$$A_{square} = BH = 4(35) = 140m$$

$$A_{triangle} = \frac{1}{2}BH = \frac{1}{2}(4)(60) = 120m$$

$$A_{total} = 120m + 140m$$

$$Displacement = 260m$$

Area – the "other" basic graph model

Let's use our new model again, but for our equation for *acceleration*.

$$Area = Bh$$

$$Acceleration = \frac{Velocity}{Time} = \frac{\Delta v}{\Delta t} or$$

$$Velocity = Time x Acceleration$$

$$\Delta v = \Delta t a$$

What does this mean?

Who gets to play the role of the base? **Time**

Who gets to play the role of the height? **Acceleration**

What kind of graph is this?

An Acceleration vs. Time graph (acceleration = y-axis & time = x-axis)

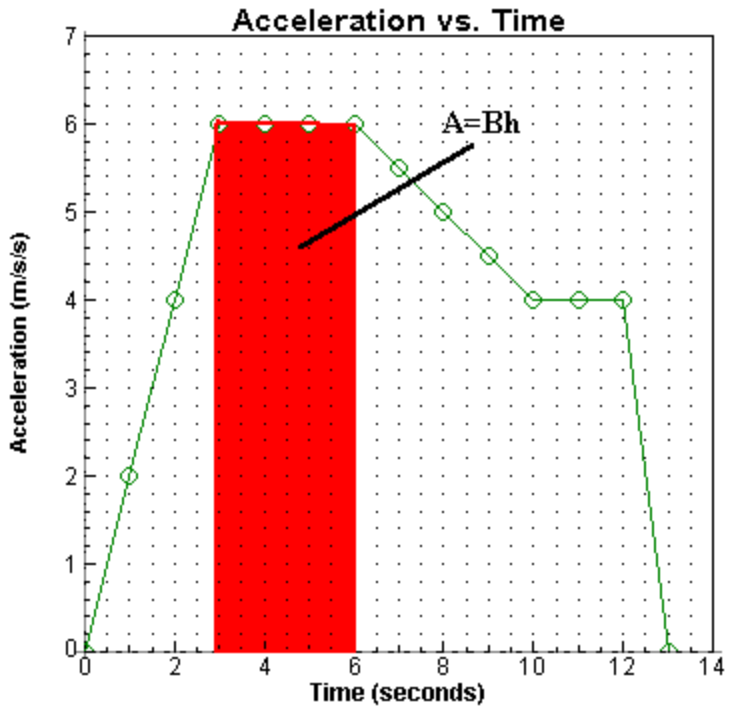
Who gets to play the role of the Area? **The Velocity**

Acceleration vs. Time Graph

What is the velocity during the time interval t = 3 and t = 6 seconds? Find the Area!

$$A = Bh \rightarrow \Delta v = ta$$

$$\Delta v = (3)(6) = 18 m/s$$



Summary

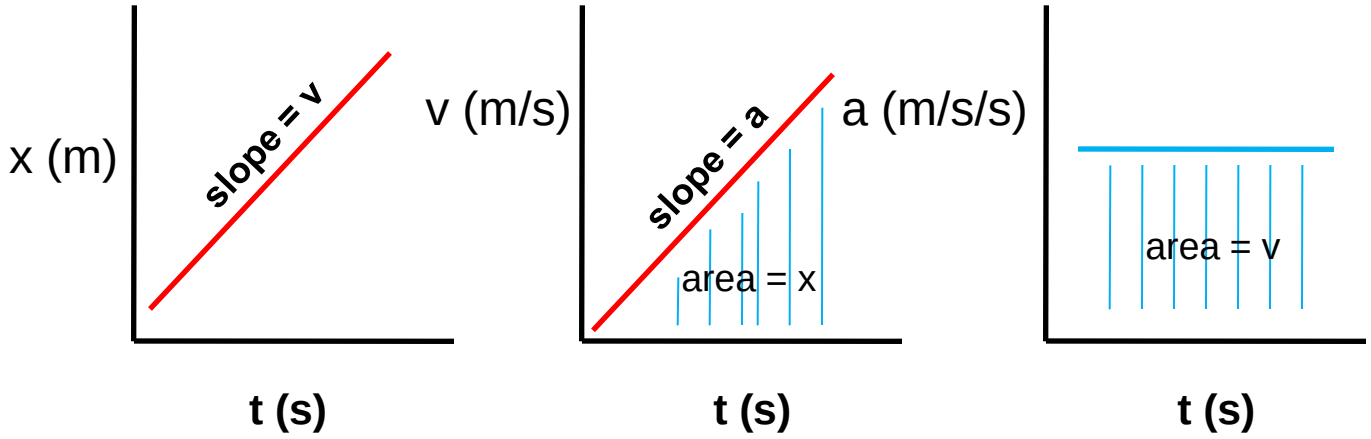
There are 3 types of MOTION graphs

- Displacement(position) vs. Time
- Velocity vs. Time
- Acceleration vs. Time

There are 2 basic graph models

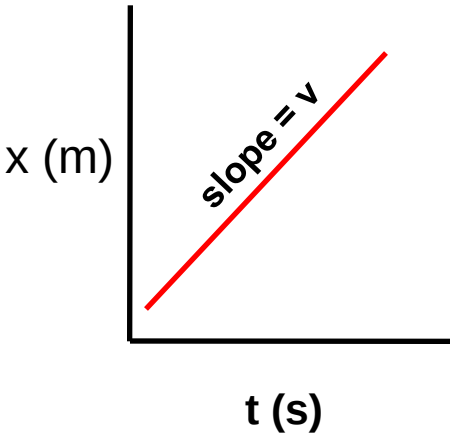
- Slope
- Area

Summary



Comparing and Sketching graphs

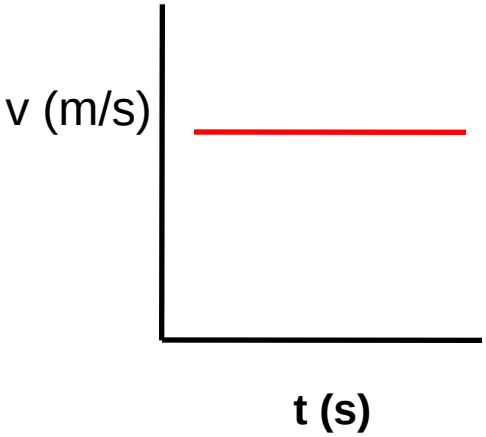
One of the more difficult applications of graphs in physics is when given a certain type of graph and asked to draw a different type of graph



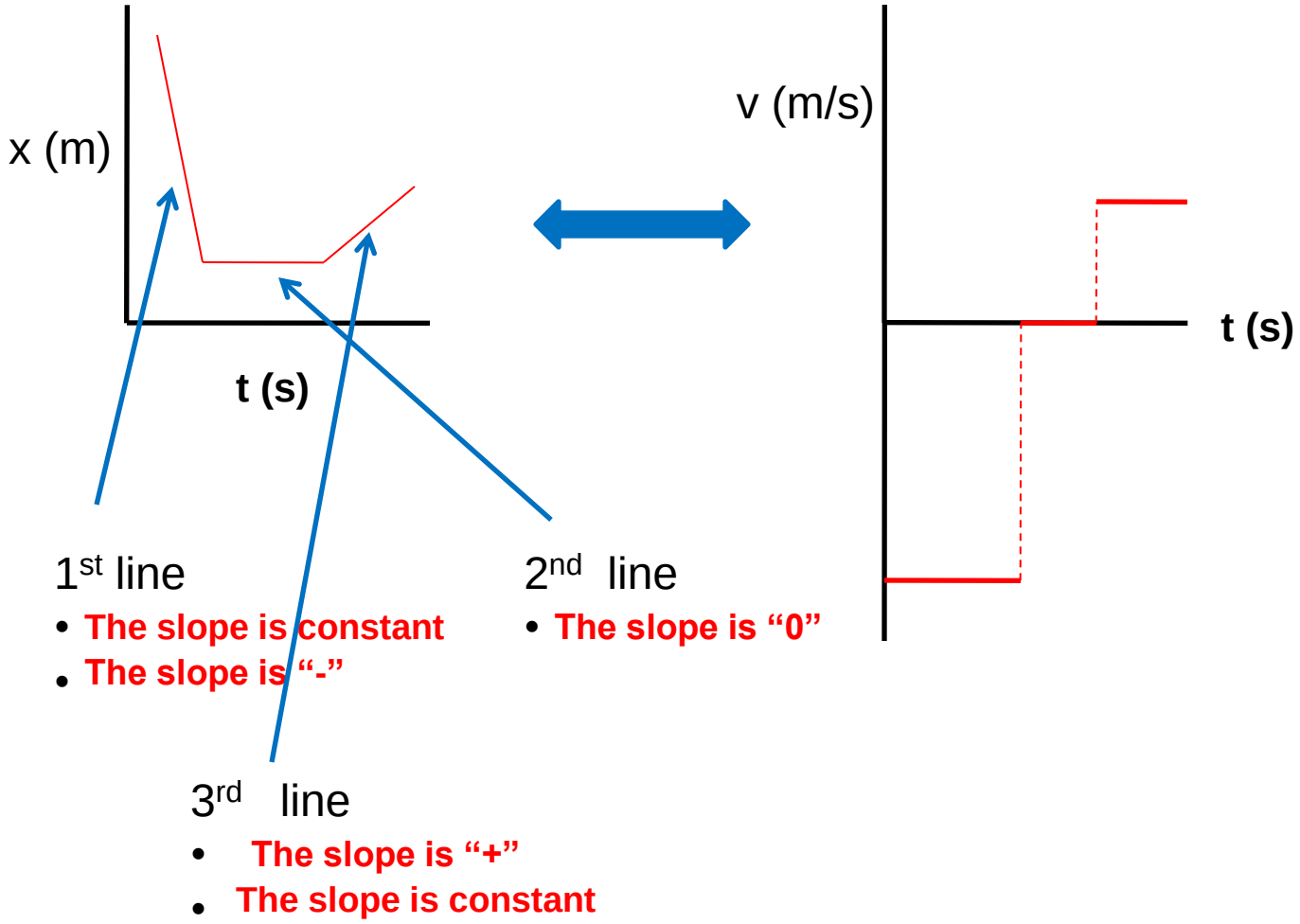
List 2 adjectives to describe the SLOPE or VELOCITY

1. The slope is CONSTANT
2. The slope is POSITIVE

How could you translate what the SLOPE is doing on the graph ABOVE to the Y axis on the graph to the right?



Example


x (m)
t (s)
v (m/s)
t (s)
1st line
• The slope is constant
• The slope is "-"
2nd line
• The slope is "0"
3rd line
• The slope is "+"
• The slope is constant

Example – Graph Matching

What is the SLOPE(a) doing?

The slope is increasing