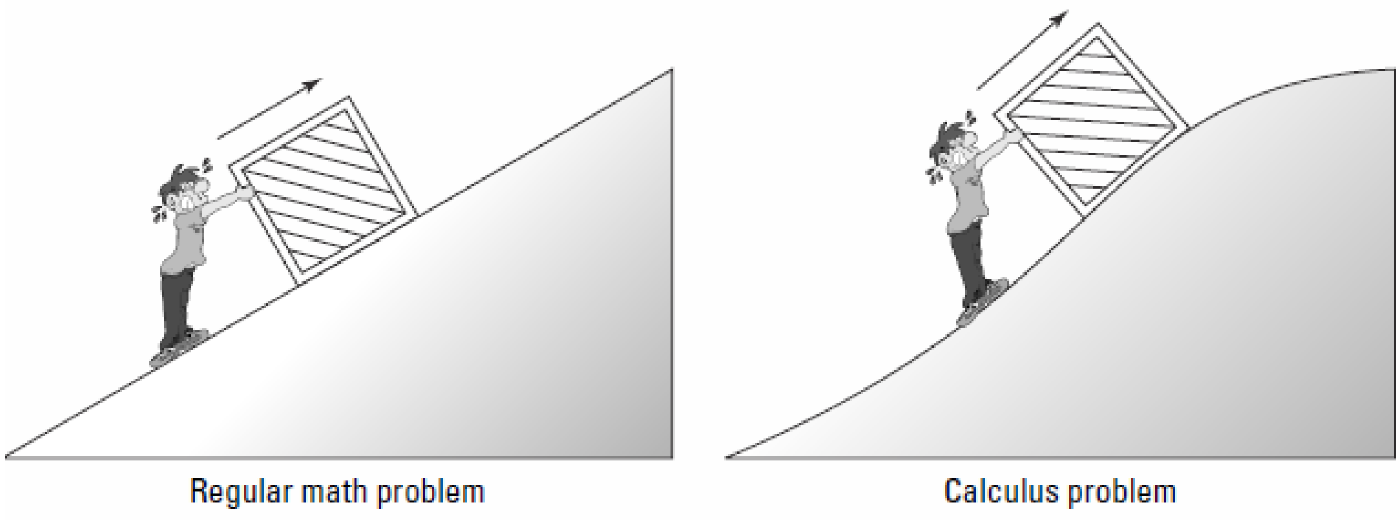
The Basics of Physics with Calculus

AP Physics C

What is calculus?

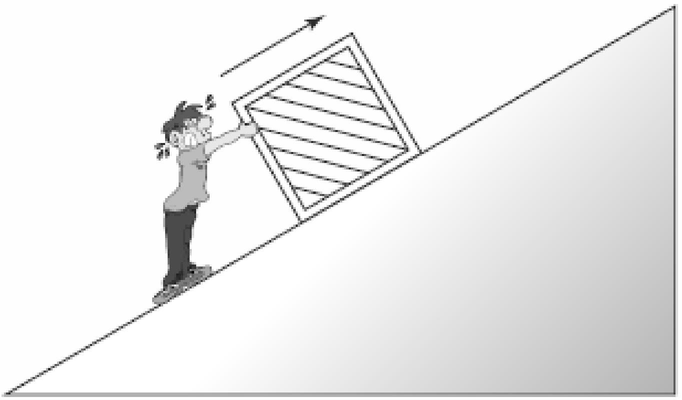
Calculus is simply very advanced algebra and geometry that has been tweaked to solve more ***sophisticated*** problems.



Question: How much energy does the man use to push the crate up the incline?

The “regular” way

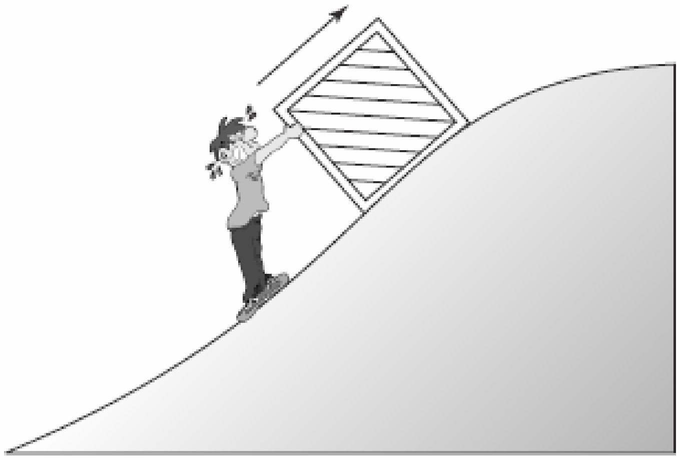
For the straight incline, the man pushes with an *unchanging* force, and the crate goes up the incline at an *unchanging* speed. With some simple physics formulas and regular math (including algebra and trig), you can compute how many calories of energy are required to push the crate up the incline.



Regular math problem

The “calculus” way

For the curving incline, on the other hand, things are constantly *changing*. The steepness of the incline is *changing* — and not just in increments like it’s one steepness for the first 10 feet then a different steepness for the next 10 feet — it’s *constantly changing*. And the man pushes with a *constantly changing* force — the steeper the incline, the harder the push. As a result, the amount of energy expended is also *changing*, not every second or every thousandth of a second, but *constantly changing* from one moment to the next. That’s what makes it a calculus problem.

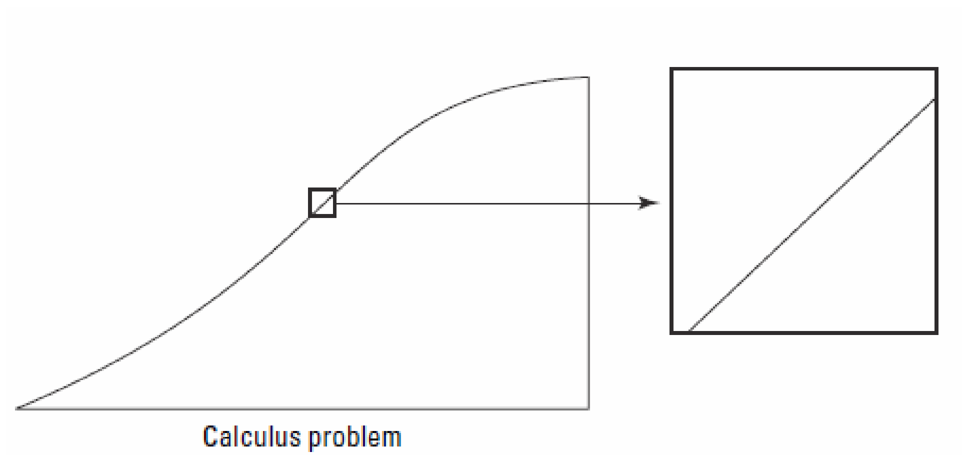


Calculus problem

What is calculus?

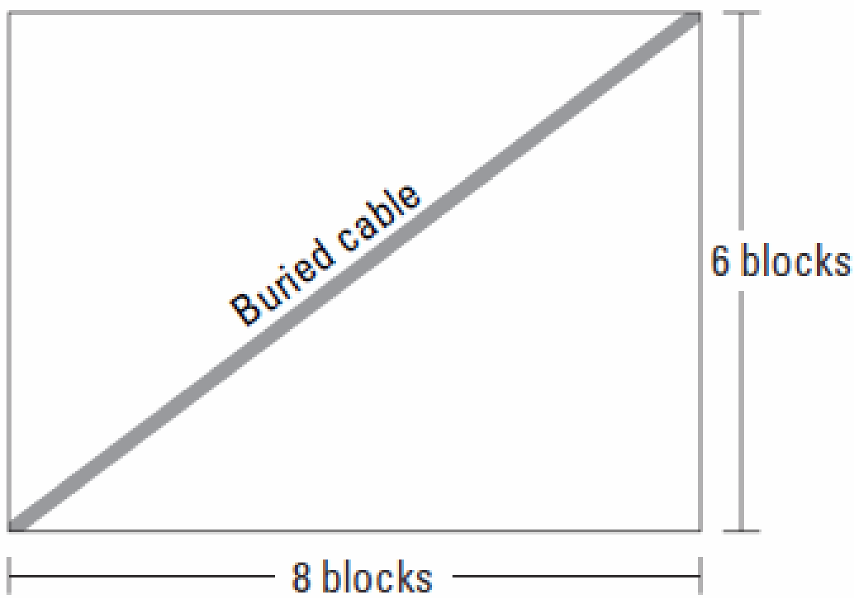
It is a mathematical way to express something that is ……CHANGING! It could be anything??

But here is the cool part:

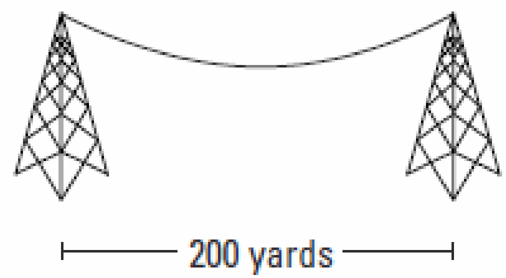


Calculus allows you to **ZOOM** in on a small part of the problem and apply the “regular” math tools.

“Regular” math vs. “Calculus”

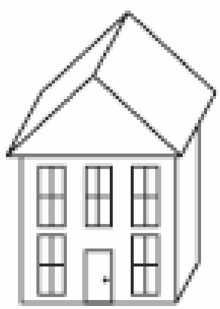


Regular math problem:
How long is the cable?

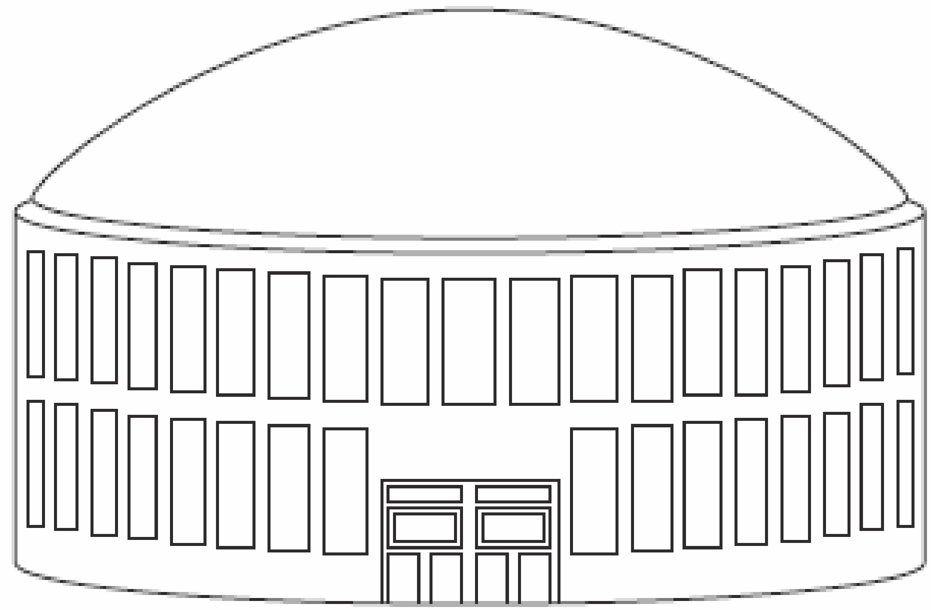


Calculus problem:
How long is the cable?

“Regular” math vs. “Calculus”

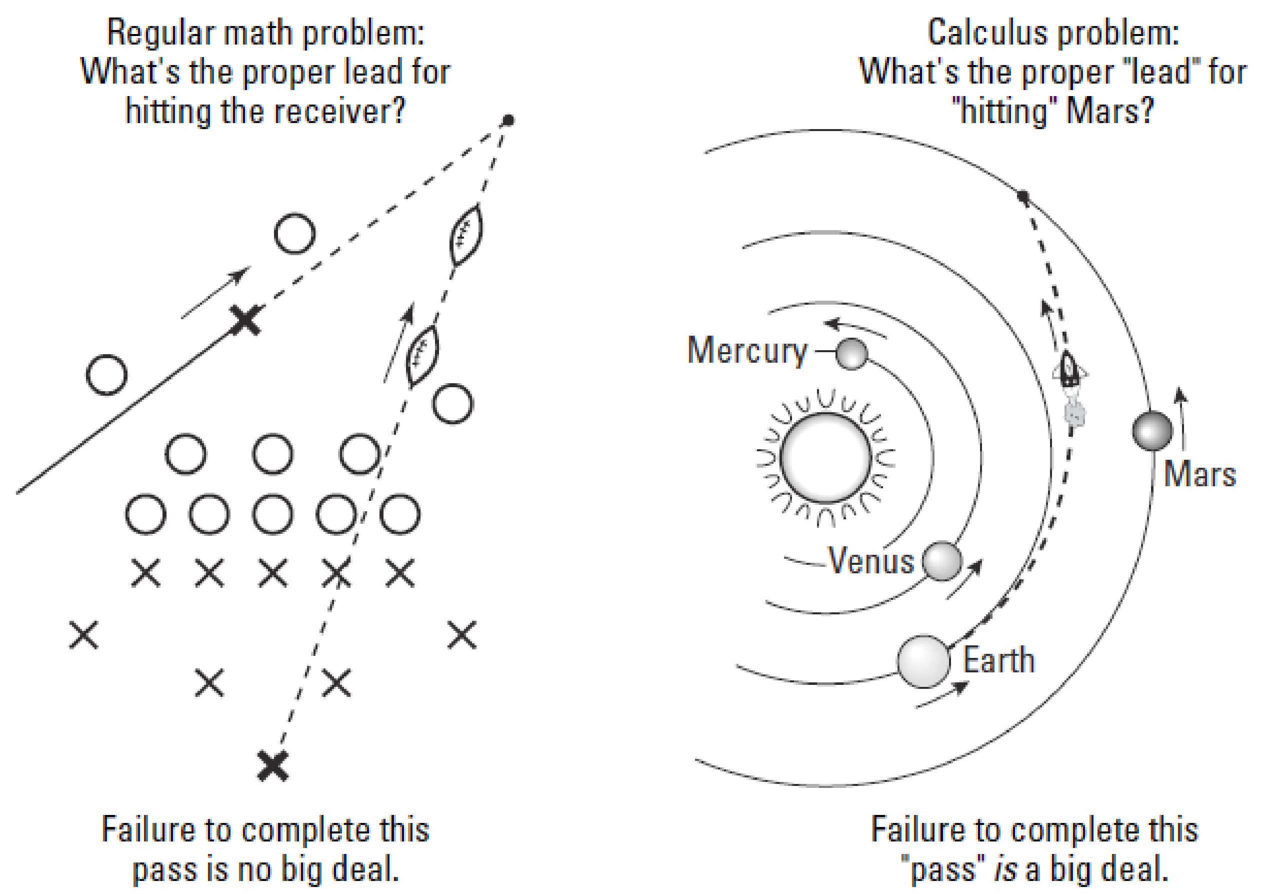


Regular math problem:
What's the roof's area?



Calculus problem:
What's the dome's area?

“Regular” math vs. “Calculus”



Learn the lingo!

Calculus is about "rates of change".

A ***RATE*** is anything divided by time.

CHANGE is expressed by using the Greek letter, Delta, Δ.

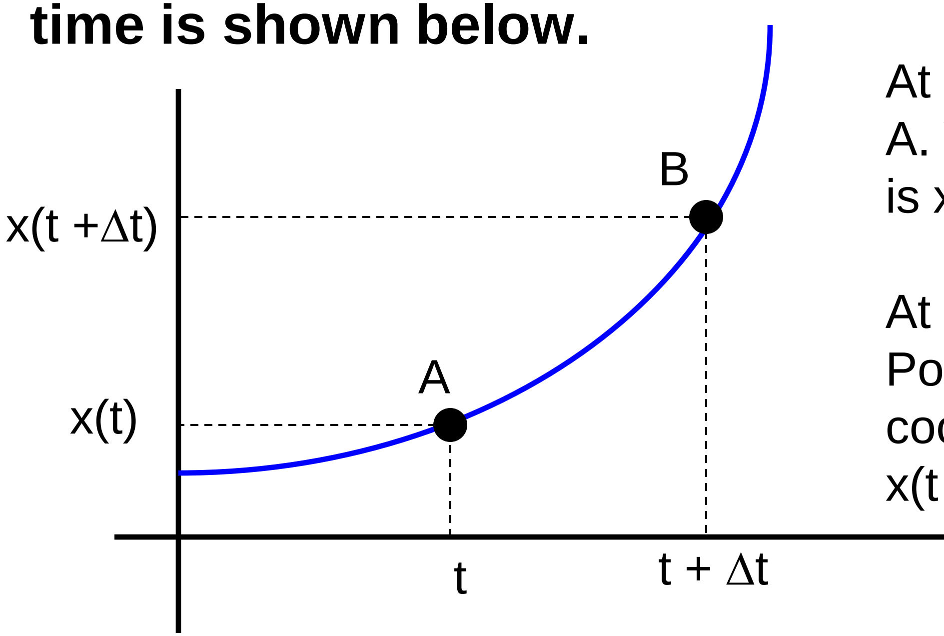
For example: Average ***SPEED*** is simply the "RATE at which DISTANCE changes".

$$\bar{s} = \frac{\Delta d}{\Delta t}$$

The Derivative…aka….The SLOPE!

Since we are dealing with quantities that are changing it may be useful to define WHAT that change actually represents.

Suppose an eccentric pet ant is constrained to move in one dimension. The graph of his displacement as a function of time is shown below.

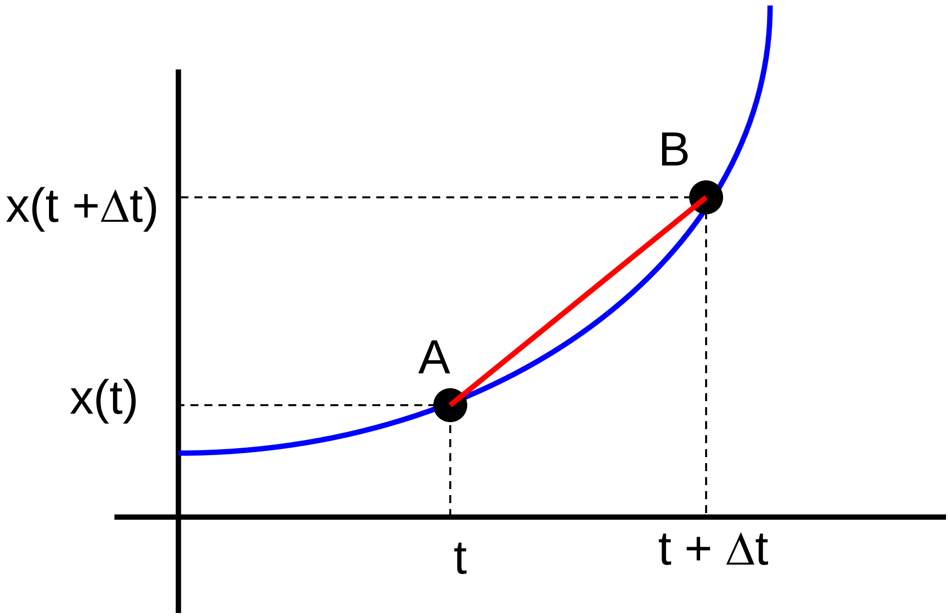


At time ***t***, the ant is located at Point A. While there, its position coordinate is x(t).

At time ***(t+ Δt)***, the ant is located at Point B. While there, its position coordinate is
x(t + Δt)

The secant line and the slope

Suppose a secant line is drawn between points A and B. Note: The slope of the secant line is equal to the rise over the run.

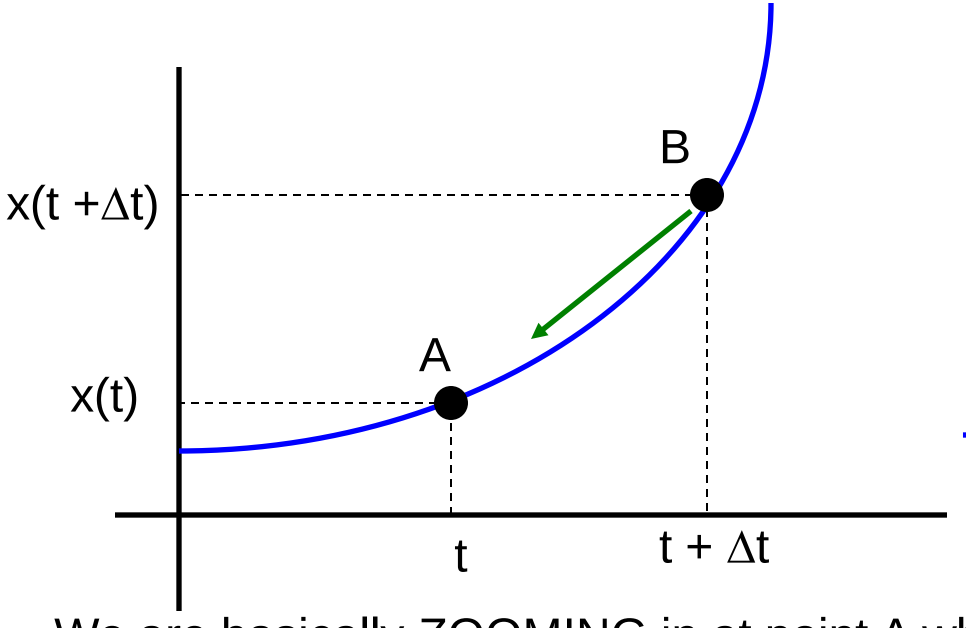


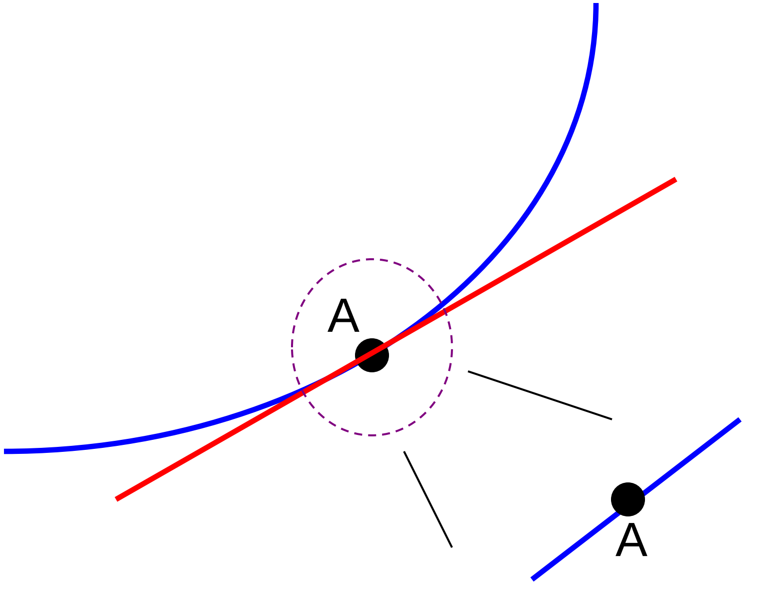
$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x(t+\Delta t) - x(t)}{(t+\Delta t) - t} = \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

The “Tangent” line

READ THIS CAREFULLY!

- **If we hold POINT A fixed while allowing Δt to become very small. Point B approaches Point A and the secant approaches the TANGENT to the curve at POINT A.**





We are basically ZOOMING in at point A where upon inspection the line “APPEARS” straight. Thus the secant line becomes a **TANGENT LINE**.

The derivative

Mathematically, we just found the slope!

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x(t+\Delta t) - x(t)}{\Delta t} = \text{slope of secant line}$$

$$\lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \text{slope of tangent line}$$

Lim stand for "LIMIT" and it shows the delta t approaches zero. As this happens the top numerator approaches a finite #.

This is what a derivative is. A derivative yields a NEW function that defines the rate of change of the original function with respect to one of its variables. In the above example we see, the rate of change of "x" with respect to time.

The derivative

In most Physics books, the derivative is written like this:

$$\frac{d(x)}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Mathematicians treat **dx/dt** as a **SINGLE SYMBOL** which means find the derivative. It is simply a mathematical operation.

The bottom line: The derivative is the slope of the line tangent to a point on a curve.

The MEANING?

For example, if t = 2 seconds, using $x(t) = kt^3=(1)(2)^3=$ 8 meters.

The derivative, however, tell us how our DISPLACEMENT (x) changes as a function of TIME (t). The rate at which Displacement changes is also called VELOCITY. Thus if we use our derivative we can find out how fast the object is traveling at t = 2 second. Since dx/dt = $3kt^2=3(1)(2)^2=$ 12 m/s

$$\frac{d(kt^3)}{dt} = 3kt^2$$

Bringing it together

- Think of a displacement vs time graph … the slope is the ____.
- The derivative of position (displacement) is _______.
- Think of a velocity vs time graph … the slope is the ____.
- The derivative of velocity is ______.

THERE IS A PATTERN HERE!!!!

- Now if I had done the previous example with **kt^2**, I would have gotten **$2t^1$**
- • Now if I had done the above example with **kt^4**, I would have gotten **$4t^3$**
- • Now if I had done the above example with **kt^5**, I would have gotten **$5t^4$**

Let’s cheat…..Forgive us Mr. Campbell!

$$If\_ x(t) = kt^n, then:$$

$$\frac{dx}{dt} = nkt^{n-1}$$

$$If\_ x(t) = 20t^3, then:$$

$$\frac{dx}{dt} = 60t^2$$

$$If\_ f(x) = 5t^3 + 6t + 7, then:$$

$$\frac{dx}{dt} = 15t^2 + 6$$

$$If\_ f(x) = 2t^6 + 7t^4 + 4t + 2, then:$$

$$\frac{dx}{dt} = 12t^5 + 28t^3 + 4$$

Common calculus derivative rules

$$-Ake^{-kt} = \frac{d(Ae^{-kt})}{dt}$$

$$k\cos(kt) = \frac{d[\sin(kt)]}{dt}$$

$$-k\sin(kt) = \frac{d[\cos(kt)]}{dt}$$

$$k(\frac{1}{t}) = \frac{d[\ln(kt)]}{dt}$$

$$\frac{k}{t} = kt^{-1} = \frac{d[kt^{-1}]}{dt} = (-1)kt^{-2} = \frac{-k}{t^2}$$